Adaptive Fractional-order Control for Synchronization of Two Coupled Neurons in the External Electrical Stimulation

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ABSTRACT

This paper addresses synchronizing two coupled chaotic FitzHugh–Nagumo (FHN) neurons with weakly gap junction under external electrical stimulation (EES). To transmit information among coupled neurons, by generalization of the integer-order FHN equations of the coupled system into the fractional-order in frequency domain using Crone approach, the behavior of each coupled neuron relies on its past behavior and the memorized system can be a better fit for the neuron response. An adaptive fractional-order controller based on the Lyaponuv stability theory was designed to synchronize two neurons electrically coupled with gap junction in EES. The proposed controller is also robust to the inevitable random noise such as disturbances of ionic channels. The simulation results demonstrate the effectiveness of the control scheme.

1. Introduction

Synchronization of electrical coupled ensemble neurons plays an important role for processing of biological information in mammalian neural systems. Synchronization of chaos is a phenomenon that may occur when two, or more, dissipative chaotic systems are coupled. In biological neural systems, chaotic synchronization means two or more coupled neurons exhibit various behaviors such as periodic, quasi-periodic and chaotic rhythms. Investigation of chaotic neuronal synchronization, for understanding of the neural circuit functioning and the brain information processing, became one of the widely studied problems in the field of neuroscience (Rehan, Hong, & Aqil, 2011). Over the past decade many brain researchers have studied the influence of external electrical stimulation (EES) like deep brain stimulation as a therapy method in cognitive disorders such as Parkinson's disease, epilepsy and dystonia (Aqil, Hong, & Jeong, 2012; Moaddy, Radwan, Salama, Momani, & Hashim, 2012). A gap junction in neural systems is an electrical synapse that is capable to transmit information among coupled neurons. Through gap junction, two neurons can communicate with each other and thus, in cognitive disorders, the decrement of its strength, can lead to communication pathways between the neurons are in trouble. Considerable theoretical and experimental efforts were devoted to the study of the gap-junctions response to EES (Aqil, Hong, & Jeong, 2012). Mathematical and computational modeling of neuron in biological systems is the simplest solution to study the transient behavior of each neuron in system under EES. From 1952, different models were presented and addressed in the literatures (Y. Chen, Bauer, Burmeister, Rupp, & Mikut, 2007; Ibarz, Casado, & Sanjuán, 2011; Izhikevich, 2003; Maeda & Makino, 2000; Tanaka, Morie, & Aihara, 2006). These models are essential for understanding the action potential, and offer predictions that may be tested against experimental data, providing a stringent test of

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a theory. The most important and accurate of the presented models is the Hodgkin–Huxley model. Although the Hodgkin–Huxley model may be a simplification of a realistic nervous membrane as existed in nature, its complexity inspired several even more simplified models. In the previous studies, some researchers used FitzHugh – Nagumo (FHN) model to study the synchronization control of two electrically coupled chaotic neurons under EES. Without control, identical coupled neurons can eventually synchronize only when the coupling strength is above a certain critical value (H. Yu et al., 2012).

The theory of non-integer order derivatives dates back to the Leibniz's note in his list to L'Hopital, on 30 September 1695, in which the meaning of the derivative of order one half is discussed (Podlubny, 1999). The study of such a fractional order differ-integral lies in a mathematical field, named fractional calculus. One of the advantages of the fractional-order system is that, each added parameter (fractional-order parameter) enriches the system performance through increasing one degree of freedom efficiently. Descriptive differential equations of a Fractional-Order System (FOS) are defined in terms of non-integer differ-integrals. FOS is adopted increasingly in diverse sciences such as physics (Yajima & Yamasaki, 2012), mathematics (Kazem, Abbasbandy, & Kumar, 2012), and engineering (Baños, Cervera, Lanusse, & Sabatier, 2011; Li, Chen, & Ahn, 2011; Wong, Li, & Leung, 2012) for modeling and control purposes.

Computationally, many control strategies were developed to achieve synchronization of electrically coupled neurons (Aqil et al., 2012; Che et al., 2011; Motallebzadeh, Jahed Motlagh, & Rahmani Cherati, 2012; Rehan et al., 2011; Xiao-Li, 2011; H. Yu et al., 2012). Yu (H. Yu et al., 2012) used an adaptive backstepping sliding model controller to achieve chaos synchronization of two coupled FHN neurons. They combined both the merits of adaptive backstepping control and sliding mode control to synchronize and approximate the disturbances of ionic channels of the dynamical system. Motallebzadeh (Motallebzadeh et al., 2012) for synchronization of chaotic systems with different orders, designed the module control by means of two different methods (active control vs. optimal control). Because of in most of the practical applications, the exact values of the model parameters may not be definite; they used a modified recursive least square algorithm to identify and estimate the unknown parameters of the model. Similar work was conducted (Xiao-Li, 2011). It should be noted that most of the above studies in order to synchronize the two coupled FHN neurons, used the integer-order differential models describing the signal transmission across axons in neurobiology systems. Modeling and synchronization of chaotic systems in neural networks and biological systems pave the way for an opening and recent topic on FOS (L. Chen, Chai, Wu, Ma, & Zhai, 2013; SABATIER & FARGES, 2012; Yu, Hu, & Jiang, 2012). Recently, synchronizing chaotic FHN model based on FOSs without control was investigated (Moaddy et al., 2012). On the other hand, study of chaotic synchronization of fractional-order coupled neurons attracted some attention due to its applications. Bhalekar (Moaddy et al., 2012) used simple linear feedback controller to synchronize the chaotic behavior of the novel Lorenz-like chaotic system. Based on FOS, Chunlai (Moaddy et al., 2012) applied adaptive schemes for control and synchronization of the three-dimensional fractional-order Chen and Liu chaotic systems. In these works, authors analyzed the computational chaotic systems based on known parameters without considering the disturbances and uncertainties of control parameters of the model to control and synchronize the chaotic system. Moaddy (Moaddy et al., 2012) represented the non-standard finite difference scheme together with the Grünwald–Letnikov discretization process for finding the numerical solutions of the chaotic synchronization with a gap junction of coupled neurons of fractional-order. But this paper represents a frequency domain method of adopting FOS in control of FHN chaotic system, called Crone approach. FOS is calculated by the use of continuous-time approximation of fractional-order operator, so the fractional system is capable to simulate differential equations of the FHN coupled neurons without discretizing of the continuous-time model. The main objective of this paper is to design an adaptive control methodology to synchronize electrically coupled neurons with weak coupling, given a fractional-order FHN model. A Model Reference Adaptive System (MRAS) is formulated by fractional derivative of Lyaponuv function candidate with respect to time. This control law includes two corrective signals and synchronizes the system, quickly.

The rest of this paper is organized as follows. Section 2 introduces mathematical preliminaries to FOS and cable fractional-order model of the FHN coupled neurons and the design of an adaptive controller based on Lyaponuv function. The simulation results of the synchronization of coupled neurons under EES are presented in section 3. Finally, discussion and conclusion are drawn in section 4.

2. Methods

In this section, for the purpose of self-containing, a brief introduction on fractional-order operator and its approximation is presented. Then cable model of FHN coupled neurons based on fractional-order is introduced and finally the applied control methodology is designed.

2.1. Fractional-Order Differ-Integral Definition

Fractional-order differ-integral operator denoted by , is a combination of differentiation and integration used in fractional calculus. Definition of this operator is as followed:

$$ {}_{a}D_{t} = \begin{cases} \overline{\quad} \\ dt & , \quad 0 \\ 1 & , \quad q = 0 \\ \int (d \quad) & , \quad q \quad 0 \end{cases} \qquad (1) $$

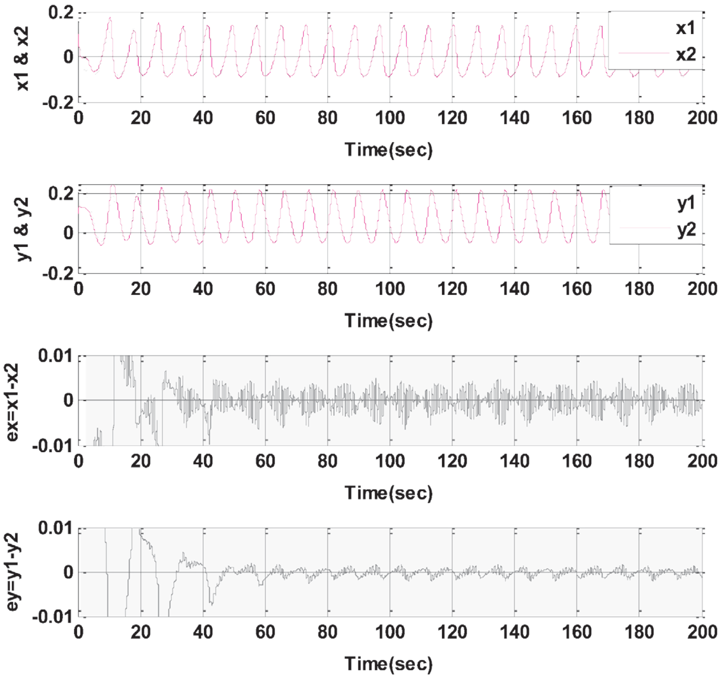

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Figure 1. Time courses of the coupled chaotic FHN neuron system: X_1 nd X_2

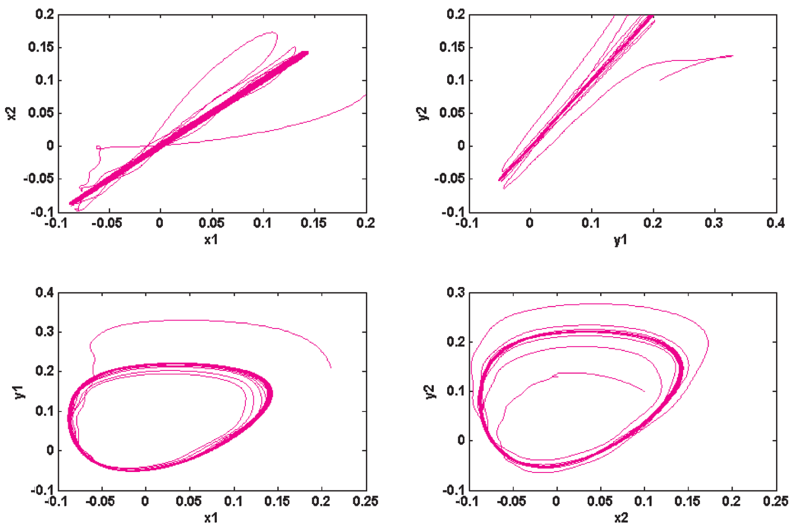

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Figure 2. Phase portraits of the neural system under complete synchronization with $\alpha = \beta = 0.85$

It must be mentioned that there are different definitions used for fractional-order operator. Herein, for the purpose of brevity, these time domain definitions are not given, but frequency domain presentation of fractional-order operator is stated as:

$$L\{D^q\} = s^q \qquad (2)$$

where stands for Laplace transformation. Frequency response of fractional-order operator (in light of equation (2)) is given by:

$$|G(j\omega)| = |j^q \omega^q| = |\omega^q| = \omega^q \Rightarrow 20\log_{10}|G(j\omega)| = 20q.\log_{10}\omega$$
$$\arg[G(j\omega)] = \arg[j^q \omega^q] = \arg[j^q] = q\frac{\pi}{2} \qquad (3)$$

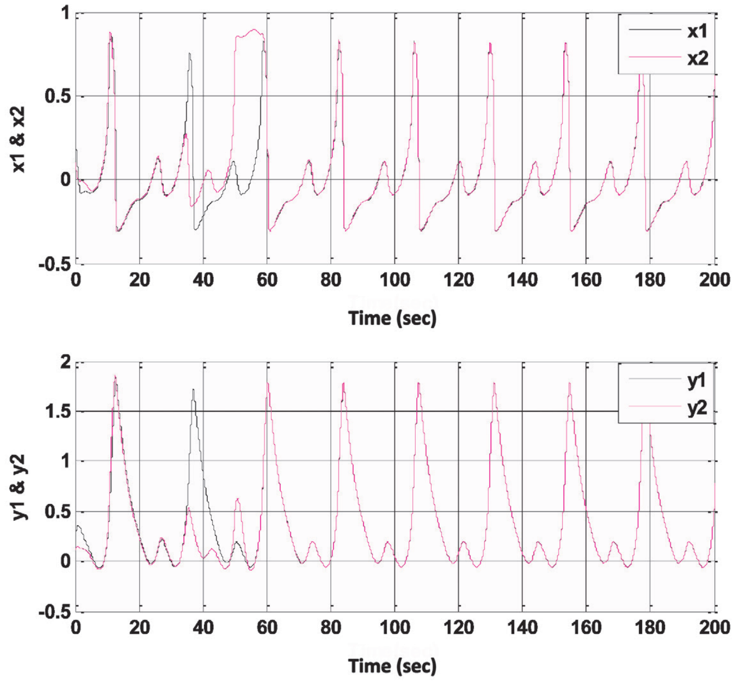

Figure 3. The state response curves before and after applying the controller when system and controller parameters are chosen to be $\alpha = \beta = 0.95$, $D = 0.01$, $k_1 = 0$, and $k_2 = 0$.

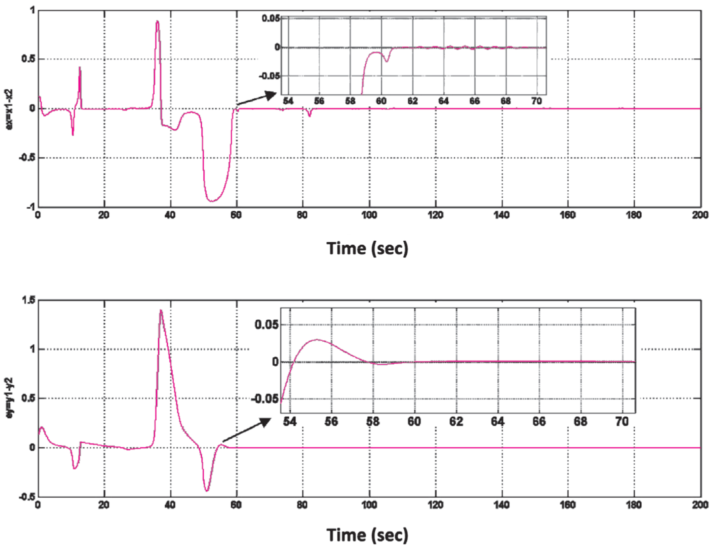

Figure 4. The error response curves before and after control when system and controller parameters are chosen to be $\alpha = \beta = 0.95$, $D = 0.01$, $k_1 = 0$, and $k_2 = 0$.

in frequency domain, practical implementation of a fractional-order operator requires an integer-order approximation. There is a vast literature on FOS realizations (Barbosa & Machado, 2006; Oustaloup, Levron, Mathieu, & Nanot, 2000; Tseng, 2001). FOS realization can be performed by discrete time or continuous time approximations. The Crone approximation is a well-known continues time one and will be used in what follows (Oustaloup et al., 2000). Crone approximation is stated as:

$$s^{v} \approx k\prod_{n=1}^{N}\frac{1+\frac{s}{\omega_{zn}}}{1+\frac{s}{\omega_{pn}}} \qquad (4)$$

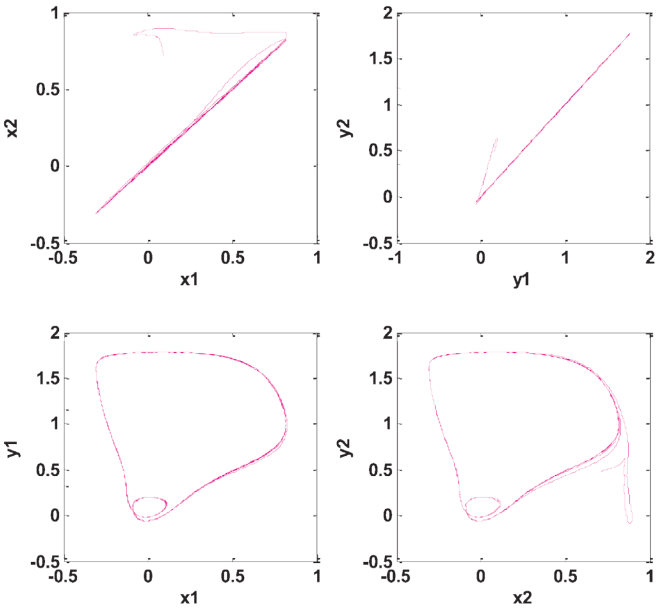

Figure 5. The phase diagrams of system (9) when system and controller parameters are chosen to be $\alpha=\beta=0.95$, $D=0.01$, $k_1=0$, and $k_2=0$.

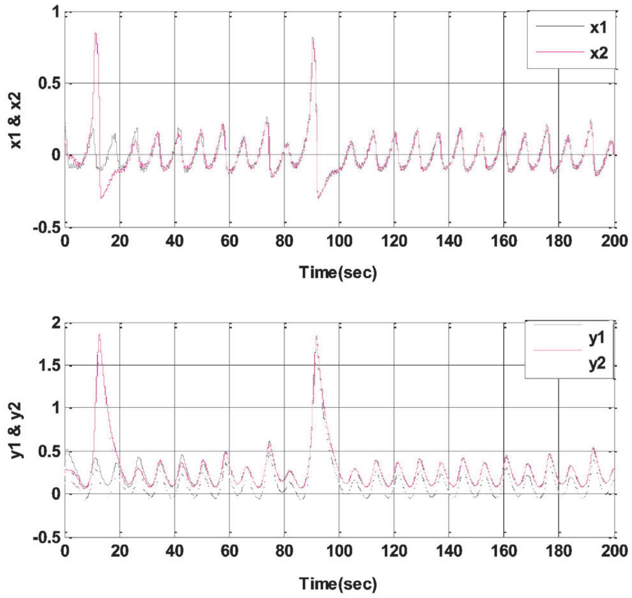

Figure 6. The state response curves before and after applying the controller when system and controller parameters are chosen to be $\alpha=\beta=0.95$, $D=0.01$, $k_1=0$, and $k_2=0$.

Where frequencies of poles and zeros are given by:

$$\omega_{z1} = \omega_l \sqrt{\eta},\ \omega_{pn} = \omega_{zn}\alpha, n = 1...N$$
$$\omega_{zn+1} = \omega_{pn}\eta,\ n = 1...N-1 \quad (5)$$
$$\alpha = (\omega_h / \omega_l)^{\frac{v}{N}},\ \eta = (\omega_h / \omega_l)^{\frac{1-v}{N}}$$

the approximation (4) is valid in the frequency range $[\omega_l, \omega_h]$; gain k is adjusted so as (3) to be satisfied; the number of poles and zeros N is chosen arbitrarily. This technique distinguishes itself from existing realization of fractional-order FHN model in being a continuous time implementation (Moaddy et al., 2012).

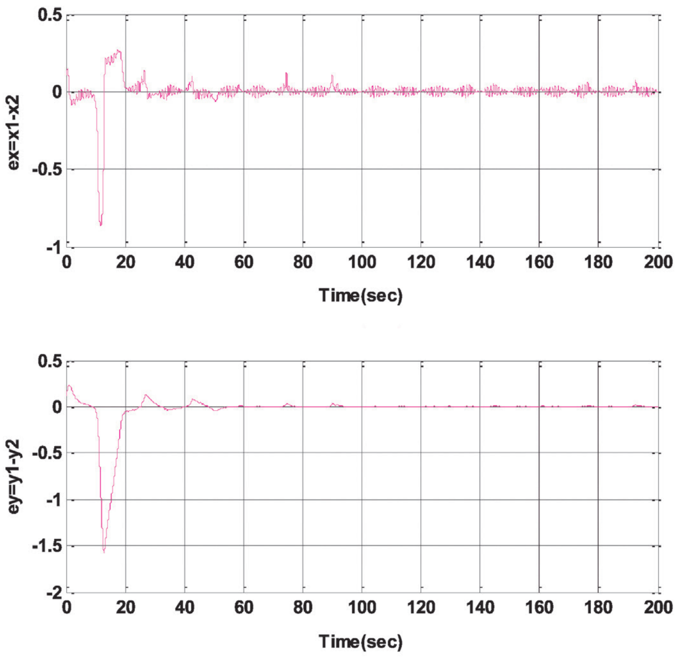

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Figure 7. The error response curves before and after applying the controller when system and controller parameters are chosen to be $\alpha = \beta = 0.95$, $D = 0.01$, $k_1 = 0$, and $k_2 = 0$.

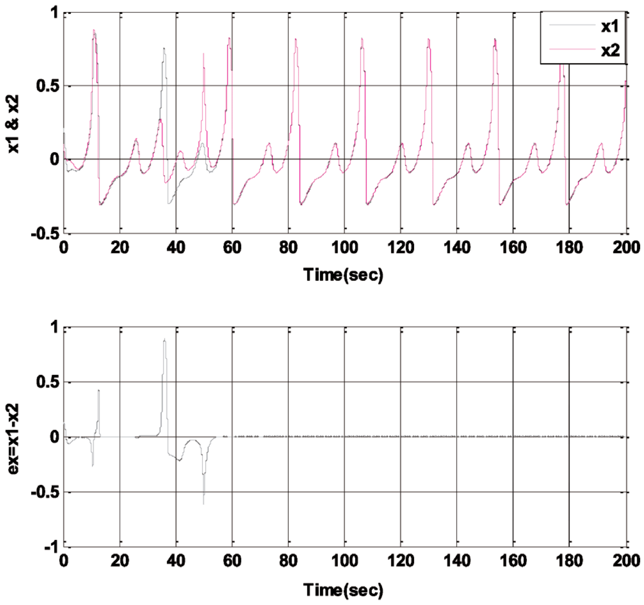

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Figure 8. The error response curves before and after applying the controller when system and controller parameters are chosen to be $\alpha = \beta = 0.95$, $D = 0.01$, $k_1 = 1$, and $k_2 = 1$.

2.2. The Nonlinear Fractional-order Model of Neuron

There are many models that are presented to study the chaotic response and synchronization of coupled virtual neurons in simulation studies (Ibarz et al., 2011; Izhikevich, 2003). One of the simplified forms of the Hodgkin and Huxley model was the one presented by FitzHugh and Nagumo in which the membrane conductance of cell is expected to be a nonlinear function of the membrane voltage, activation and inactivation of ionic channels such as Potassium and Sodium. The space-clamped model of the FHN neuron based on fractional-order is described as (Maeda & Makino, 2000):

$$\begin{aligned} \frac{d^{\alpha}X}{dt^{\alpha}} &= X(X-1)(1-rX)-Y+I_0 \\ \frac{d^{\beta}Y}{dt^{\beta}} &= bX \end{aligned} \qquad (6)$$

where X and Y are rescaled the membrane voltage and recovery variable, respectively. I_0 is the EES in form of $I_0=\frac{A}{\omega}\cos(\omega t)$ with $A, \omega=2\pi f$ the amplitude (A) and frequency (HZ), respectively, and b, r are bifurcation parameters and in complete synchronization of two coupled neurons without control were fixed, while in control process considered unknown and will be estimated adaptively. α,β denote non-integer orders of the system. Throughout this paper, the parameter A was fixed at value of 0.1. If the parameter f is chosen $0.1245<f<0.131$, the system exhibits a chaotic behavior (Che et al., 2011). In this paper, we fixed this parameter in a value of 0.128 Hz. The model of two gap junction coupled FHN neurons is considered by:

$$\begin{aligned} \frac{d^{\alpha}X_1}{dt^{\alpha}} &= X_1(X_1-1)(1-rX_1)-Y_1-g_{12}(X_1-X_2)+I_0+d_1 \\ \frac{d^{\beta}Y_1}{dt^{\beta}} &= bX_1 \\ \frac{d^{\alpha}X_2}{dt^{\alpha}} &= X_2(X_2-1)(1-rX_2)-Y_2-g_{21}(X_2-X_1)+I_0+d_2 \\ \frac{d^{\beta}Y_2}{dt^{\beta}} &= bX_2 \end{aligned} \qquad (7)$$

Where X_i *and* $Y_i\,(i=1,2)$ are state variables of two neurons in (6), respectively and $g_{12}=g_{21}=g$ is the coupling strength of gap junction. In order to evaluate the effectiveness and robustness of the proposed control scheme (see section 3.2), we add the perturbation d_1 and d_2 to the controlled coupled system, i.e. $d_1=D\sin(2\pi f_1 t)$ and $d_2=D\sin(2\pi f_2 t)$ for all the time. The Sinusoidal noise as a source of high-frequency time-varying external disturbances is usually used to simulate the ionic channel noise of neurons, which is actually the most common disturbance in neuroscience (Aqil et al., 2012; Rehan et al., 2011). If the coupling strength of the gap junction in non-fractional order , the synchronization occurs (H. Yu et al., 2012).

2.3. MRAS Controller Design for a Fractional-order Model of Electrically Coupled Neurons

Herein, fractional-order of differentiating FHN model equations is chosen to model a natural state of neurons in terms of action potential. It is assumed that 1. In (9), r and b are unknown parameters, 2. For fractional-orders, $\alpha=\beta$ and 3. Two control signals u_x and u_y are applied to the second neuron's membrane voltage and recovery variable, respectively. Hence, the differential equation of coupled system is rewritten as:

$$\begin{aligned} \frac{d^{\alpha}X_1}{dt^{\alpha}} &= X_1(X_1-1)(1-rX_1)-Y_1-g_{12}(X_1-X_2)+I_0+D\sin(2\pi f_1 t) \\ \frac{d^{\beta}Y_1}{dt^{\beta}} &= bX_1 \\ \frac{d^{\alpha}X_2}{dt^{\alpha}} &= X_2(X_2-1)(1-rX_2)-Y_2-g_{12}(X_2-X_1)+I_0+D\sin(2\pi f_2 t)+u_x \\ \frac{d^{\beta}Y_2}{dt^{\beta}} &= bX_2+u_y \end{aligned} \qquad (9)$$

The error dynamic system can be got as:

$$\begin{aligned} \frac{d^{\alpha}}{dt^{\alpha}}e_x &= \frac{d^{\alpha}X_2}{dt^{\alpha}}-\frac{d^{\alpha}X_1}{dt^{\alpha}} = X_2^2-X_1^2-e_x+(\hat{r}+\tilde{r})(X_2^2-X_1^2-X_2^3+X_1^3)-e_y+u_x \\ &-2ge_x+2D[\cos(\pi(f_1+f_2)t)\sin(\pi(f_2-f_1)t)] \\ &= X_2^2-X_1^2-e_x+(\hat{r}+\tilde{r})(X_2^2-X_1^2-X_2^3+X_1^3)-e_y+u_x-2ge_x+d \\ \frac{d^{\alpha}}{dt^{\alpha}}e_y &= \frac{d^{\beta}Y_2}{dt^{\beta}}-\frac{d^{\beta}Y_1}{dt^{\beta}} = (\hat{b}+\tilde{b})e_x+u_y, \quad \tilde{r}=r-\hat{r}, \quad \tilde{b}=b-\hat{b} \end{aligned} \qquad (10)$$

The Lyaponuv function candidate is chosen as:

$$V(e_x,e_y)=\begin{bmatrix} e_x & e_y \end{bmatrix}\begin{bmatrix} e_x \\ e_y \end{bmatrix}+\tilde{r}^2+\tilde{b}^2=e^T e+\tilde{r}^2+\tilde{b}^2 \qquad (11)$$

Hence, the fractional derivative of V of order α with respect to time, along the trajectories of Eq. (10) is

$$\begin{aligned} \frac{d^{\alpha}}{dt^{\alpha}}V &= e^T e^{(\alpha)}+\tilde{r}\tilde{r}^{(\alpha)}+\tilde{b}\tilde{b}^{(\alpha)}+\sum_{k=1}^{\infty}\frac{\Gamma(1+\beta)}{\Gamma(1+k)\Gamma(1-k+\beta)}e_x^{(k)}e_x^{(\beta-k)} \\ &+\sum_{k=1}^{\infty}\frac{\Gamma(1+\beta)}{\Gamma(1+k)\Gamma(1-k+\beta)}e_y^{(k)}e_y^{(\beta-k)}+\sum_{k=1}^{\infty}\frac{\Gamma(1+\beta)}{\Gamma(1+k)\Gamma(1-k+\beta)}\tilde{r}^{(k)}\tilde{r}^{(\beta-k)}+\sum_{k=1}^{\infty}\frac{\Gamma(1+\beta)}{\Gamma(1+k)\Gamma(1-k+\beta)}\tilde{b}^{(k)}\tilde{b}^{(\beta-k)} \\ &= e^T e^{(\alpha)}+\tilde{r}\tilde{r}^{(\alpha)}+\tilde{b}\tilde{b}^{(\alpha)}+R \end{aligned} \qquad (12)$$

Where R consists of the remainder of fractional-order derivative. Since the sign of R is unknown (M. O. Efe, 2008; M. Ö. Efe & Kasnakoḡlu, 2008), let the following bound conditions hold:

$$|R| < B_1, \ |d| < B_2 \tag{13}$$

Where d is the disturbance or noise in ionic channels given in (10). It is worth mentioning that d has such a small value that its effect can be neglected in control system design (Wang & Ren, 2011). Thus, it is reasonable to put an upper boundary on d. By choosing control laws (u_x and u_y) and update laws ($\dot{\hat{b}}$ and $\dot{\hat{r}}$) to be:

$$\begin{aligned} u_x &= -\hat{r}\left(X_2^2 - X_1^2 - X_3^2 + X_1^3\right) - X_2^2 + X_1^2 + e_y \\ u_y &= -\hat{b}e_x - e_y \\ \frac{d^\alpha \hat{b}}{dt^\alpha} &= e_x e_y + k_1 \operatorname{sgn}(\tilde{b}) \\ \frac{d^\alpha \hat{r}}{dt^\alpha} &= e_x\left(X_2^2 - X_1^2 - X_2^3 + X_1^3\right) + k_2 \operatorname{sgn}(\tilde{r}) \end{aligned} \tag{14}$$

And using these laws in (12), following equation will be obtained:

$$\frac{d^\alpha}{dt^\alpha}V = -(1+2g)e_x^2 - e_y^2 - k_1\tilde{b}\operatorname{sgn}(\tilde{b}) - k_2\tilde{r}\operatorname{sgn}(\tilde{r}) + de_x + R \tag{15}$$

If and are chosen so as to , the fractional derivative of V will be negative and the convergence will be achieved.

3. Results

3.1. Complete Synchronization of Two Coupled Neurons without Control

In this section, numerical simulations using MATLAB Simulink (THE MATHWORKS, R2012), and a solver ode45 (Dormand-Prince) with relative tolerance 1e-3 carried out for synchronization of the coupled FHN neuron systems with the coupling strength of and initial conditions as:

$$X_1(0) = 0.21, \quad Y_1(0) = 0.21, \quad X_2(0) = 0.1, \quad Y_2(0) = 0.1 \tag{8}$$

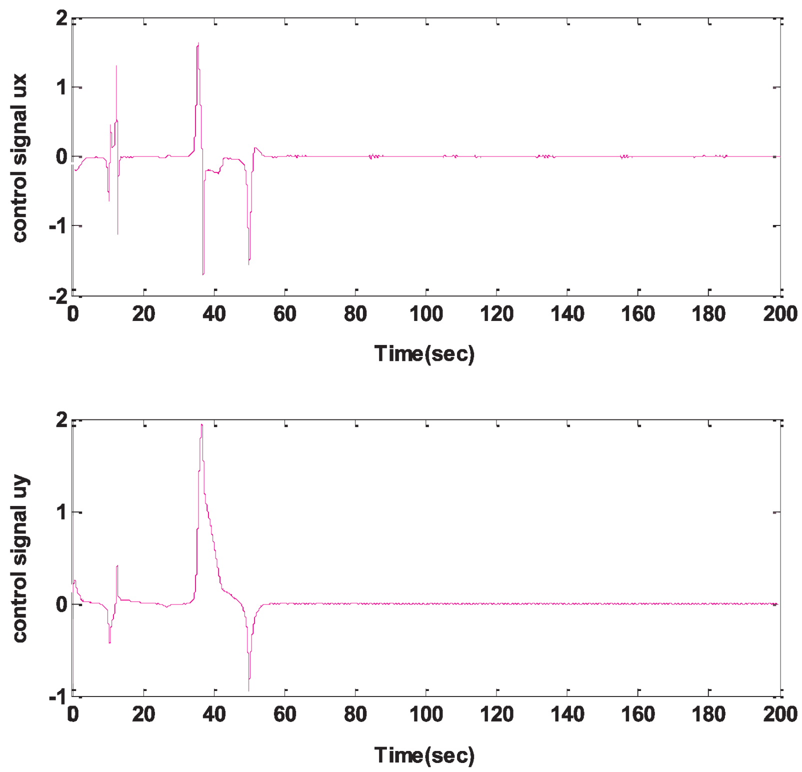

Figure 9. Control signals when system and controller parameters are chosen to be $\alpha = \beta = 0.95$, $D = 0.01$, $k_1 = 1$, and $k_2 = 1$.

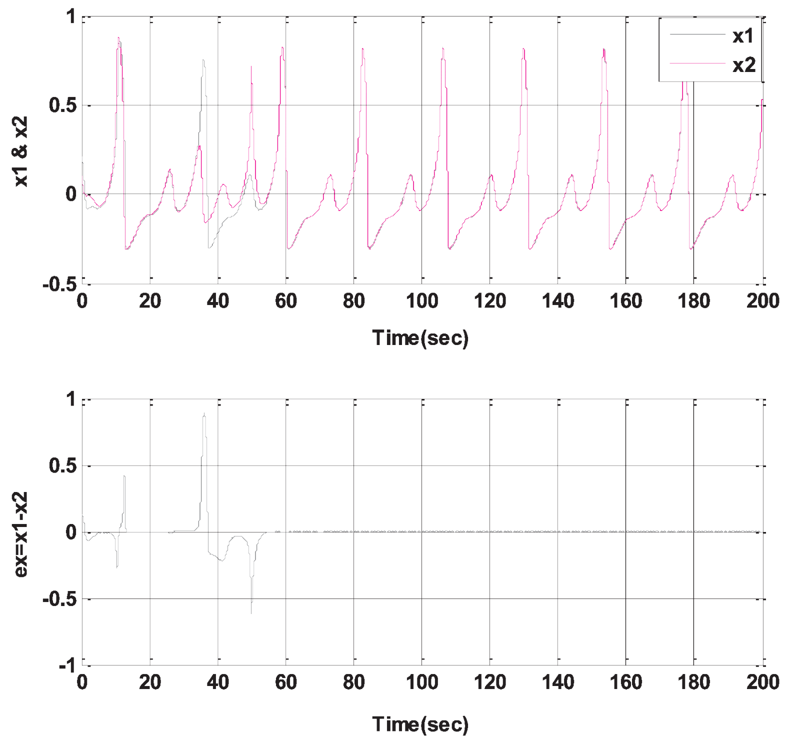

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Figure 10. The error response curves before and after applying the controller when system and controller parameters are chosen to be $\alpha=\beta=0.95$, $D=0.01$, $k_1=100$, and $k_2=100$.

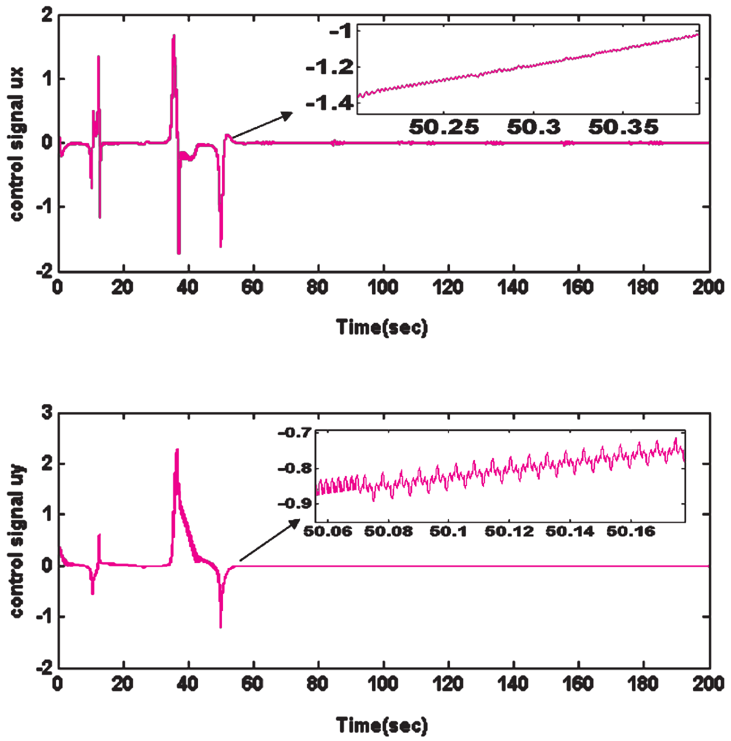

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Figure 11. Control signals when system and controller parameters are chosen to be $\alpha=\beta=0.95$, $D=0.01$, $k_1=100$, and $k_2=100$.

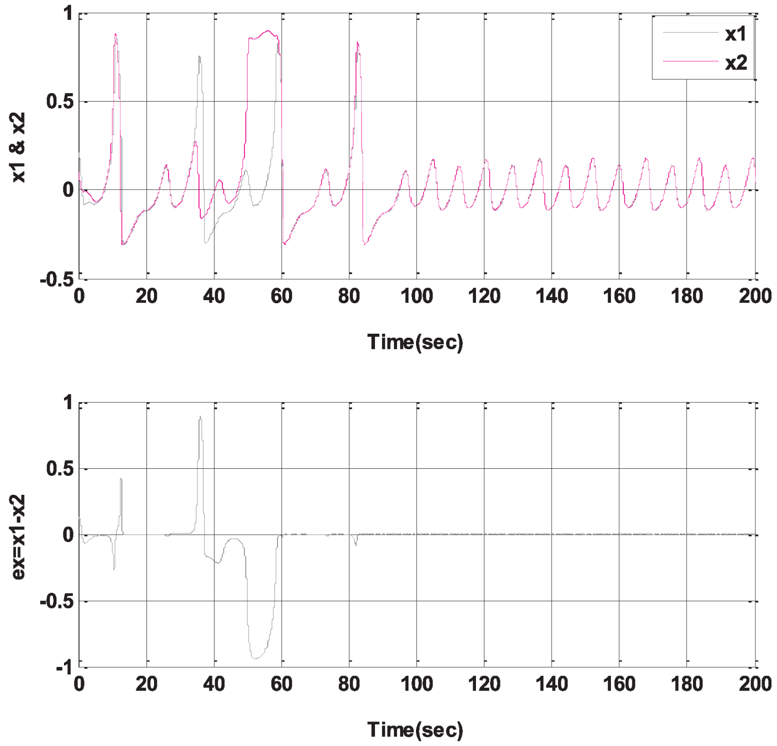

Figure 12. The state of X_i (i = 1, 2) and its error response curves with the decreament of parameters b and r, when system and controller parameters are chosen to be $\alpha = \beta = 0.95$, $D = 0.01$, $k_1 = 0$, and $k_2 = 0$.

Since the dynamics of individual FHN neuron without coupling should be chaotic (H. Yu et al., 2012), we fixed parameters A, f, r, and b at values 0.1, 0.1271, 10 and 1 (without control), respectively. The simulation results are depicted in Figures 1 and 2. Figure 1 shows the state error signal responses. With $\alpha = \beta = 0.85$ the exact synchronization is got. When the fractional-order decreases, the synchronization becomes faster. Figure 2 shows the phase portraits of the system on planes of $X_i - Y_i (i = 1, 2)$. With decrement of α, β to a value of 0.85 phase locking of the states imply identical behaviors of the two neurons ($X_1 - X_2$ and $Y_1 - Y_2$).

3.2. MRAS Controller Design

Simulation results were obtained based on the following assumptions 1. The total time range is chosen to be 200 seconds, 2. In time instance 50 control signals were applied, 3. Gap junction value is , 4. Amplitude of the ionic channel disturbances were $D = 0.01$ and $f_1 = 1$, $f_2 = 1.1\ Hz$, and 6. With trial and error k_1 and k_2 were selected to be zero for the case study. When fractional order values were fixed in $\alpha = \beta = 0.95$, State response curves illustrated in figure 3. The respective error curves of the system before and after adopting controller are displayed in figure 4. Figure 4 demonstrates that after almost 61 seconds, the controller (14) can synchronize two neurons and resist the random disturbance efficiently. Referring to the magnified transient response of the stabilizing error in figure 4, the state errors rapidly converge to zero in finite time, once the control signal is bring to function. Figure 5 depicts the phase diagram of the system in the time interval $50 < t < 200$ sec. As shown in figure 5, by applying controller, the fractional-order neurons are synchronized profoundly. If amplitude of ionic canel disturbances become 10 times greater ($D = 0.1$) and fractional-orders $\alpha = \beta = 0.9$ are selected, by applying the controller, simulation results will be that of shown in figures 6 and 7. By decreasing the values of fractional-orders from 0.95 to 0.9, e.g., by introducing more memory to system, the amplitude of state decreases. Nevertheless, increasing the amount of disturbances 10 times greater, two neurons are synchronized by applying controller due to the fact of capable of having more memory. When parameters k_1 and k_2 are selected to be nonzero numbers for instance, 1, simulation results shown in figures (8) and (9) demonstrate that error signal would be approximately zero and control signal chattering will not occur. However, since function sign (.) exists in (14), by increasing the value of these parameters, the influence of sign function will be dominant and in turn error signals will reach to zero at the cost of control signal chattering. For instance, chattering phenomenon seen in figure 11, using $k_1 = k_2 = 100$, while there are slight improvements in error signals as shown in figure 10. If the parameters b and r decrease from their nominal values to 0.8 and 8, respectively (or 20% decrement) at time $100 \sec$, as shown in figure 12, the synchronization of the system will not be influenced.

4. Discussion And Conclusions

In this paper, to synchronize two coupled FHN neurons with weakly gap junction, an adaptive control was provided. One of the simplified forms of the Hodgkin and Huxley model was the one presented by FitzHugh and Nagumo. In this model, the membrane potential and activation or inactivation of ionic channels were replaced with fast and slow variables as a two-dimensional nonlinear system. In order to transmit information among coupled neurons, the behavior of a neuron relies on its past behavior. To consider this memory, a fractional-order approach was applied to memorize the coupled system. Moaddy (Moaddy et al., 2012) and his colleagues implemented fractional-order systems by discrete-time Grünwald–Letnikov definition. This discretization methodology may slightly increase the complexity of system calculation. Herein, in contrast with existing works, a continuous-time approximation of fractional-order systems named Crone was performed. Crone is a frequency domain implementation of FOS in which there is no requirement to discretize of system differential equations. In our work, the fractional-order of the model was kept a suitable constant fraction. This gives rise to maintain chaotic dynamics and fast variable (or action potential) peak does not decrease suddenly.

Over the past ten years, several authors developed many different control methods to synchronize electrically coupled neurons (Aqil et al., 2012; Che et al., 2011; Motallebzadeh et al., 2012; Rehan et al., 2011; Xiao-Li, 2011; H. Yu et al., 2012). In this research, we designed an adaptive control methodology to synchronize the state variables of chaotic nonlinear systems, given a fractional-order model of neurons. An adaptive fractional-order controller, is capable to synchronize two neurons electrically uncoupled in EES effectively, considering that the membrane voltage and recovery variable of the coupled system can be controlled with only one controller. This kind of control system uses a Lyaponuv function candidate which is fractional-order differentiated with respect time. Moreover, (Moaddy et al., 2012) authors in their models did not consider the disturbances of ionic channels of simulated neuron membrane and used the fixed parameters to study the performance and robust analysis of their control methodologies. Since the bifurcation parameters of the model (*b & r*) are unknown, so with adaptive estimating of these parameters, our proposed controller is robust to the inevitable random noise such as disturbances of ionic channels and uncertainties of the dynamic of the model. Moreover, by increasing the values of parameters k_1, k_2, the chattering phenomenon occurred, while the magnitude of error signals decreased very slightly.

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